When you copy a system of equations from the text, include the curly-brace. The text does not show this notation but I want all students to use it. This notation shows the original state of the equations in the system. That is, before any changes have been made.

The two methods of solving a system of two equations are called “substitution” and “elimination”.

The substitution method involves solving one equation for a letter and then substituting for that letter in the second equation.

See example 1 page 158. The second equation is already solved for x so we simply replace the x in the first equation with (y + 1) and then simplify. We get a value for y. We put that value into one of the original equations. I now call that equation the “used” equation. We simplify to get the x value. Now we substitute **both** the x value and the y value in the “unused” equation to check.

Example 2. The first equation is easily solved for y. That value is then substituted into the second equation. We put the value of 4 in for x in one of the original equations (now called the used equation) and solve for y. Finally we put x and y into the “unused” equation and simplify as a check. Your work should look like this:

$$\begin{cases} 2x + y = 6 \\ 3x + 4y = 4 \end{cases}$$

$$y = -2x + 6$$

$$3x + 4(-2x + 6) = 4$$

$$3x - 8x + 24 = 4$$

$$-5x = -20$$

$$x = 4$$

$$2(4) + y = 6$$

$$8 + y = 6$$

$$y = -2$$

$$(4, -2)$$

We write the solution to intersection problems as coordinates.

Generally we will not show a check but if we did, it would look like this:

$$\begin{cases} 3(4) + 4(-2) = 4 \\ 12 - 8 = 4 \\ 4 = 4 \\ (4, -2) \end{cases}$$

Notice the curly-brace on the original equations. **Notice** that the work is written *v e r t i c a l l y*

Notice the solution is written as a coordinate pair. This is the conclusion of the problem.

The second method described in this section of the book is called elimination. This requires that both equations are written in the same order and that the equal signs are aligned.

Generally we multiply one or both of the equations by numbers that would, when the two equations are added together, eliminate at least one of the letters.

If that process eliminates both of the letters **and** the resultant equation is **true**, we know one equation was an multiple of the other and that **there are infinite solutions** because there is no single point of intersection.

If that process eliminates both of the letters **and** the resultant equation is **false**, we know that the lines were parallel and there was **no solution** to the system because the lines were parallel.

Example page 161

$$\begin{cases} 2x - 3y = 0 \\ -4x + 3y = -1 \end{cases}$$

Elimination Method

Notice that *if* we simply added the top line to the bottom line we would *eliminate* **at least one of the letters**.

$$-2x = -1$$
$$x = \frac{1}{2}$$

$$2\left(\frac{1}{2}\right) - 3y = 0$$
$$1 - 3y = 0$$
$$1 = 3y$$
$$\frac{1}{3} = y$$

$$-4\left(\frac{1}{2}\right) + 3\left(\frac{1}{3}\right) = -1$$
$$-2 + 1 = -1$$
$$-1 = -1$$

$$\left(\frac{1}{2}, \frac{1}{3}\right)$$

Sometimes the equations are not so easily added.

$$\begin{cases} 2x+3y=17 \\ \frac{1}{8}x+\frac{4}{9}y=8 \end{cases}$$

If an equation is true in the first place, then multiplying both sides of the equation by a number will not change the fact that the relationship is true. We use this fact on both of our equations. Notice that multiplying the bottom equation by $8 \cdot 9$ will remove the franctions. After those multiplications we have:

$$2x+3y=17$$
$$9x+32y=576$$

We **cannot add the two equations together** *yet* because we would **not eliminate at least one variable**.

Consider further multiplications: Multiply the top by – 9 and the bottom by 2.

$$-9(2x+3y=17)$$
$$2(9x+32y=576)$$

Gives us

$$-18x-27y=-153$$
$$18x+64y=1152$$

Add the two lines together

$$37y=999$$
$$y=27$$

$$2x+3(27)=17$$
$$2x+81=17$$
$$2x=-64$$
$$x=-32$$
$$(-32,27)$$

Section 3.3 Applications of systems of two equations

There are some relationships that are common.

- Rate **times** Time **equals** Distance.
- Number of items **times** Value Each **equals** Total Value.
- Rate of completion of a job per hour **times** hours **equals** amount of the job completed.
- Percent of an item **times** Amount of a solution containing that item **equals** amount of that item.

The most difficult of the items is the "mixture" type of problem.

Example 3, page 168.

We see that we have two variety of teas.

Let $t =$ amount of oolong tea
$a =$ amount of shaved almonds

We know we are to make 300 ounces therefore we know $t + a = 300$.

The second equation is the key to this type of problem.

We use the "Number of items **times** Value Each **equals** Total Value" to get the total value for *each* of the items.

The value of the tea is 215 cents per ounce and we have t ounces of tea so the value of the tea is $215t$.

Likewise, we have the total value of the sliced almonds as $95a$.

Finally, when we add the two items together we end up with 300 ounces worth 185 cents per ounce.

Our equation: $$\underbrace{215t}_{\text{value of oolong tea}} + \underbrace{95a}_{\text{value of sliced almonds}} = \underbrace{300 \cdot 185}_{\text{total value of both items}}$$

Our system of equations is: $$\begin{cases} t + a = 300 \\ 215t + 95a = 300 \cdot 185 \end{cases}$$

Multiplying the top equation by – 95 and then adding to the second line, we get:
(Note: I did not multiply the 20 and the – 95 yet.)

$$\begin{aligned} -95t - 95a &= 300 \cdot (-95) \\ 215t + 95a &= 300 \cdot (185) \end{aligned}$$

$$120t = 300(90)$$

$$t = \frac{\overset{75}{\cancel{300}} \cdot \overset{3}{\cancel{90}}}{\underset{1}{\cancel{120}}^{\cancel{4}}}$$

$t = 225$ and 75 ounces of almonds

So we require 225 ounces of Oolong tea and 75 ounces of shaved almonds to make a mixture of 300 ounces worth $1.85 per ounce.

Notice that I didn't multiply the 300 and the – 95. Basically I have – 95x and + 185x that need to be added. I add them and get 90x. that is really 90 • 300 which I can reduce much easier than I could reduce 27000 with the denominator of 120.

Another view of a similar situation is example 5 page 170.

The general set up is almost the same as the previous example. The second equation may be difficult to understand:

$$\underbrace{.03G}_{\text{amount of nitrogen}} + \underbrace{.08S}_{\text{amount of nitrogen}} = \underbrace{(.06)(90)}_{\textit{Total}\text{ amount of nitrogen}}$$

This is a description of "liters + liters = liters".

The text suggests drawing pictures to illustrate some problems
(mainly R times T equals D).

This is an excellent suggestion.

A couple of example problems follow:

Example:

A printing company charged 1.9¢ per sheet of paper but 2.4¢ per sheet for paper made of recycled fibers. The bill for 150 sheets of paper was \$3.41. How many sheets of each type of paper were used?

There are two different kinds of paper. We need to be able to tell them apart.

Let p = cheaper paper
r = recycled paper

$p + r = 150$ because the total number of sheet of paper is 150.

Now we multiply the number of pieces of cheap paper time 1.9¢ (which gives us the *total* cost of the cheap paper) and the number of pieces of recycled paper times 2.3¢. When we add these two costs it must equal the total cost of all the paper. Thus:

$.019p + .023r = 3.41$

I will solve the first equation for one of the letters and then substitute into the second equation.

$p = 150 - r$

$19(150 - r) + 23r = 3410$

etc.

Example
Joe's catering service is planning a wedding reception. The bride and groom would like to serve a nut mixture containing 25% peanuts. Joe has available mixtures that are either 40% or 10% peanuts. How much of each type should be mixed to get a 20-lb mixture that is 25% peanuts?

We have two different mixtures that contain peanuts. We want to create a third mixture that has a particular percentage of peanuts.

Let f = the number of pounds of forty percent peanuts
t = the number of pounds of ten percent peanuts

$f + t = 20$ because we will have a total of 20 pounds of the third mixture.

We know the forty percent mixture has .40 • f pounds of only peanuts. It has .60•f pounds of other nuts. Likewise, the ten percent mixture has .10 • t pounds of peanuts. Finally, the third mixture is twenty five percent peanuts and will have .25 • 20 pounds of peanuts. In algebra that will look like this:

$,4f + .1t = .25(20)$ This becomes $4f + t = 5$.
etc

Mr. Mumaugh Exam # 1 Name ______________________________

Solano College Math 104 Date ______________________________

Copy original problem.

Convince ***me*** that ***you*** know the concept.

No Calculators.

I Simplify. Do not use negative exponents in final answers. (3 pts ea)

A) $-\frac{5}{7}-\left(-\frac{9}{10}\right)$ B) $-26.2-2.24$ C) $-\frac{2}{3}\cdot\left(-\frac{3}{7}\right)$ D) $\frac{-24.8}{-6.2}$

E) $12x-3(2x-8)+3$ F) $(3x^{-6}y^{-3})(8x^{-2}y^{-5})$ G) -5^{-2} H) $-3(x-3)-7(x+4)$

I) $-(4x^2y^8)^0$ J) $\left(\frac{6x^4y^{-8}}{-18y^{-6}}\right)^2$

II Solve for x. (5 pts ea)

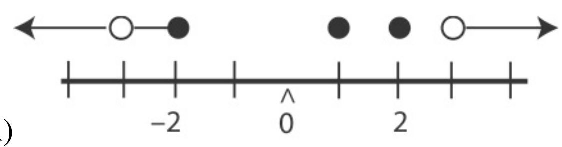
A) $2x-3=\frac{1}{3}x+\frac{1}{2}$ B) $3x-(16-5x)=3(x-2)$ C) $g=x\frac{M}{R^2}$ D) $\frac{1}{2}x+\frac{2}{3}=\frac{3}{4}+\frac{5}{6}x$

III For each problem, write the equation of the line in graphing form, draw an axis and sketch the graph. Be sure your line crosses both axes, if possible. (5 pts ea)

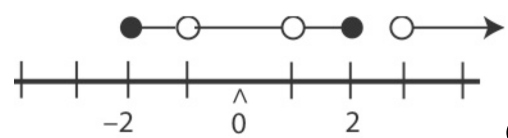
A) $-2x+y=1$ B) $5y-2x=10$ C) Line contains $(4,-1)$ and $(8,-4)$

D) Line has slope $\frac{2}{3}$ and contains $(6,3)$ E) Line contains $(15,-12)$ and is parallel to $4x-3y=9$

IV Write the intervals described by the following number lines:



A)



B) (5 pts ea)

V The pressure 100 ft beneath the ocean's surface is approximately 4 atm (atmospheres), whereas at a depth of 200 ft, the pressure is about 7 atm. (15 pts tot)

A) Write coordinates (d,a) where a would be the atmospheres at d feet. Write an equation in the form of $y=mx+b$ based on that data.

B) Use the equation you found above to find the pressure at 1,000 feet.

C) Humans can withstand up to 10 atm without assisted breathing apparatus. Use the equation you found above to find the depth a human can dive without assistance.

Extra Credit ---- 5 points ----

Alan and Betty's ages add up to 25. Betty and Charlie's ages add up to 29. Denise is 14. Twice her age is equal to the sum of Alan and Charlie's ages. Who is the youngest of the group?